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Linear, Exponential, Quadratic Model

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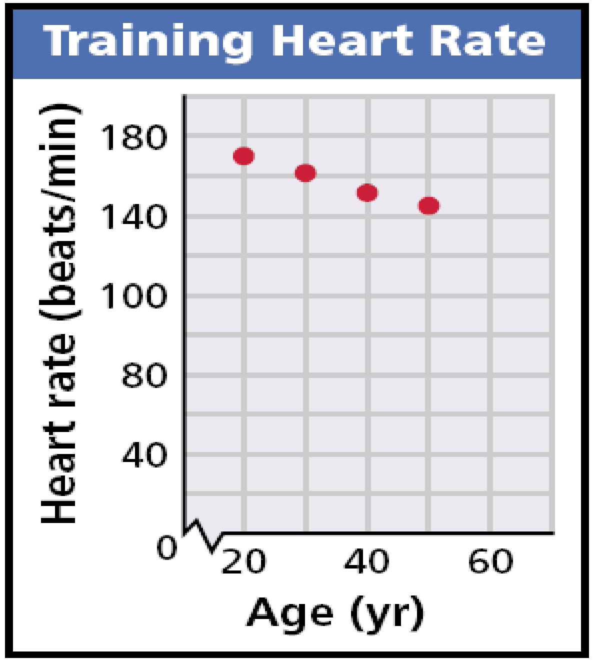
The sports data below show three kinds of variable relationships-linear, quadratic, and exponential.

Linear

Training Heart Rate

Age (yr)	Beats/min
20	170
30	161.5
40	153
50	144.5

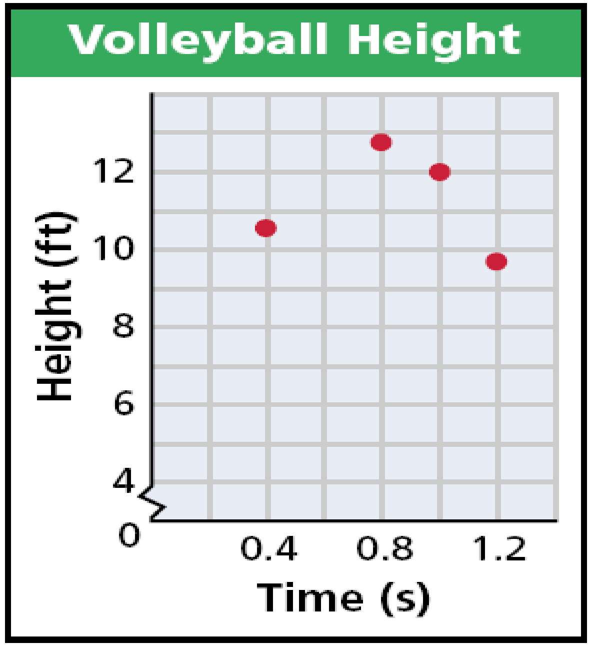


The sports data below show three kinds of variable relationships-linear, quadratic, and exponential.

Quadratic

Volleyball Height

Time (s)	Height (ft)
0.4	10.44
0.8	12.76
1	12
1.2	9.96



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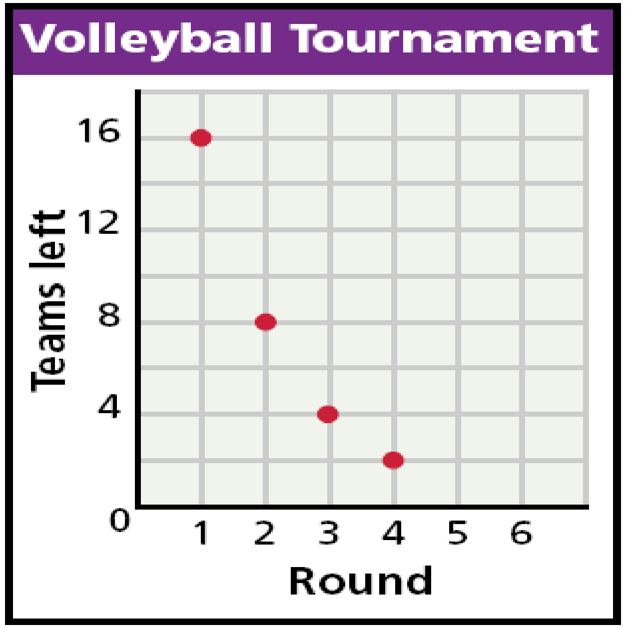
The sports data below show three kinds of variable relationships-linear, quadratic, and exponential.

Exponential

Volleyball Tournament	
Round	**Teams Left**
1	16
2	8
3	4
4	2



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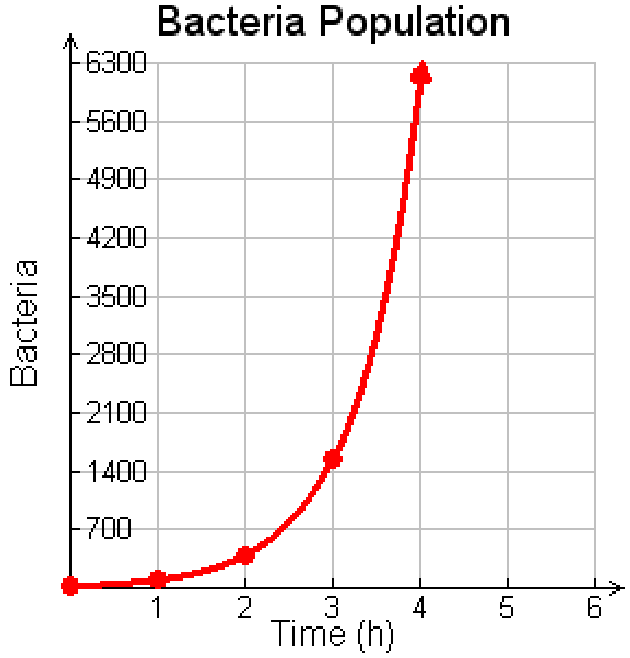
In the real world, people often gather data and then must decide what kind of relationship (if any) they think best describes their data.

Additional Example 1A: Graphing Data to Choose a Model

Graph each data set. Which kind of model best describes the data?

Time (h)	Bacteria
0	24
1	96
2	384
3	1536
4	6144



Plot the data points and connect them.

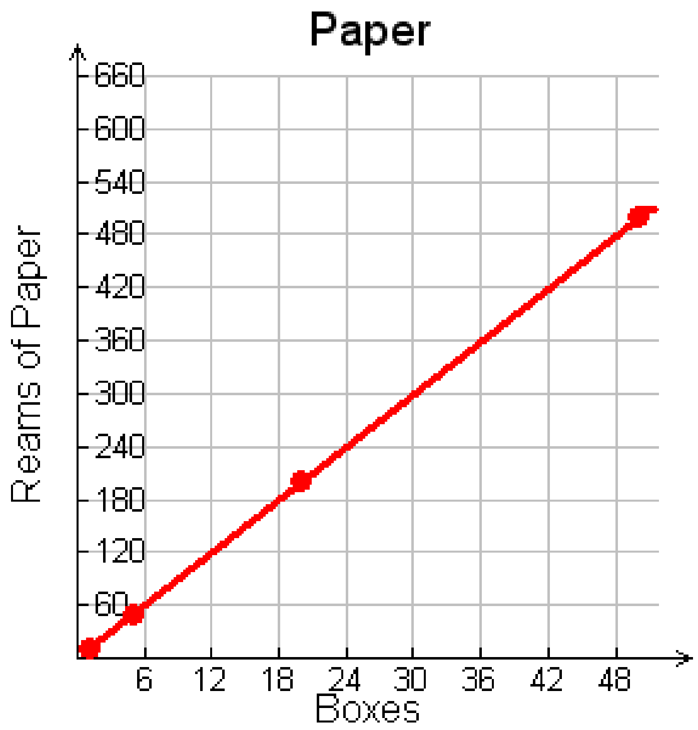
The data appear to be exponential.

Additional Example 1B: Graphing Data to Choose a Model

Graph the data set.
Which kind of model best describes the data?

Boxes	Reams of paper
1	10
5	50
20	200
50	500

The data appear to be linear.



Plot the data points and connect them.

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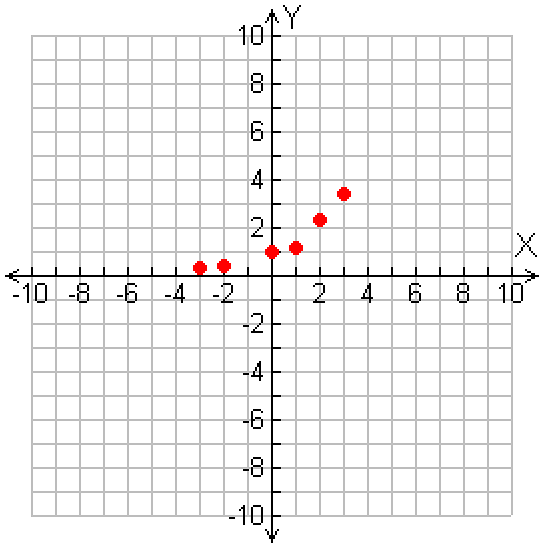
Check It Out! Example 1a

Graph the set of data. Which kind of model best describes the data?

x	*y*
–3	0.30
–2	0.44
0	1
1	1.5
2	2.25
3	3.38



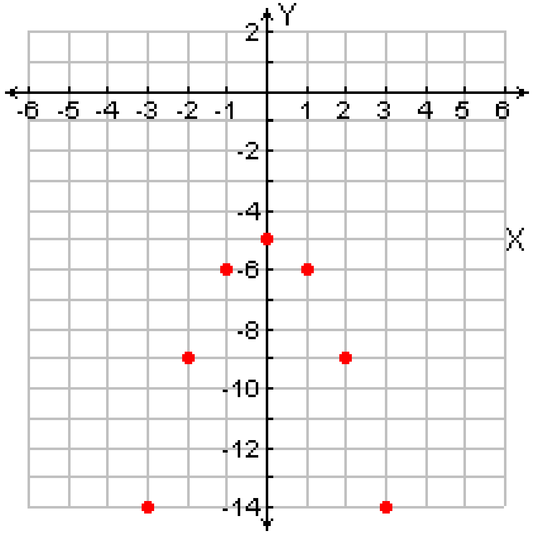
Plot the data points.

The data appear to be exponential.

Check It Out! Example 1b

Graph the set of data. Which kind of model best describes the data?

x	*y*
–3	–14
–2	–9
–1	–6
0	–5
1	–6
2	–9
3	–14



Plot the data points.

The data appear to be quadratic.

Another way to decide which kind of relationship (if any) best describes a data set is to use patterns. Look at a table or list of ordered pairs in which there is a constant change in *x*-values.

$y = -3x - 1$

	x	y	
	−2	7	
+1	−1	4	−3
+1	0	1	−3
+1	1	−2	−3
+1	2	−5	−3

Linear functions have constant **first differences**.

Another way to decide which kind of relationship (if any) best describes a data set is to use patterns. Look at a table or list of ordered pairs in which there is a constant change in *x*-values.

$$y = \frac{x^2}{2}$$

Change in x	x	y	First differences	Second differences
	−4	8		
+2	−2	2	−6	
+2	0	0	−2	+4
+2	2	2	+2	+4
+2	4	8	+6	+4

Quadratic functions have constant ***second differences***.

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Another way to decide which kind of relationship (if any) best describes a data set is to use patterns. Look at a table or list of ordered pairs in which there is a constant change in x-values.

$y = 2^x$

x	y
0	1
1	2
2	4
3	8
4	16

+1 +1 +1 +1

×2 ×2 ×2 ×2

Exponential functions have a **constant ratio**.

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Additional Example 2A: Using Patterns to Choose a Model

Look for a pattern in each data set to determine which kind of model best describes the data.

Height of Golf Ball	
Time (s)	**Height (ft)**
0	4
1	68
2	100
3	100
4	68

Time differences: + 1, + 1, + 1, + 1

First differences: + 64, + 32, 0, –32

Second differences: –32, –32, –32

For every constant change in time of +1 second, there is a constant second difference of –32.

The data appear to be quadratic.

Additional Example 2B: Using Patterns to Choose a Model

Look for a pattern in each data set to determine which kind of model best describes the data.

Money in CD	
Time (yr)	Amount ($)
0	1000.00
1	1169.86
2	1368.67
3	1601.04

+ 1, + 1, + 1

× 1.17, × 1.17, × 1.17

For every constant change in time of + 1 year there is an approximate constant ratio of 1.17.

The data appear to be exponential.

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Caution!

When solving problems like those in Example 2, be sure there is a constant change in the *x*-values.

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Check It Out! Example 2

Look for a pattern in the data set {(−2, 10), (−1, 1), (0, −2), (1, 1), (2, 10)} to determine which kind of model best describes the data.

Data (1)	Data (2)
−2	10
−1	1
0	−2
1	1
2	10

Data (1) differences: + 1, + 1, + 1, + 1

Data (2) first differences: – 9, – 3, + 3, + 9

Second differences: + 6, + 6, + 6

For every constant change of +1 there is a constant second difference of 6.

The data appear to be quadratic.

After deciding which model best fits the data, you can write a function. Recall the general forms of linear, quadratic, and exponential functions.

General Forms of Functions

LINEAR	QUADRATIC	EXPONENTIAL
$y = mx + b$	$y = ax^2 + bx + c$	$y = ab^x$

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Additional Example 3: *Problem-Solving Application*

PROBLEM SOLVING

Use the data in the table to describe how the number of people changes. Then write a function that models the data. Use your function to predict the number of people who received the e-mail after one week.

E-mail Forwarding	
Time (Days)	**Number of People Who Received the E-mail**
0	8
1	56
2	392
3	2744

1 Understand the Problem

The answer will have three parts–a description, a function, and a prediction.

2 Make a Plan

Determine whether the data is linear, quadratic, or exponential. Use the general form to write a function. Then use the function to find the number of people after one week.

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3 Solve

Step 1 Describe the situation in words.

E-mail Forwarding	
Time (Days)	**Number of People Who Received the E-mail**
0	8
1	56
2	392
3	2744

+ 1, + 1, + 1 (Time); × 7, × 7, × 7 (Number of People)

Each day, the number of e-mails is multiplied by 7.

Step 2 Write the function.

There is a constant ratio of 7. The data appear to be exponential.

$y = ab^x$	*Write the general form of an exponential function.*
$y = a(7)^x$	
$8 = a(7)^0$	*Choose an ordered pair from the table, such as (0, 8). Substitute for x and y.*
$8 = a(1)$	*Simplify. $7^0 = 1$*
$8 = a$	*The value of a is 8.*
$y = 8(7)^x$	*Substitute 8 for a in $y = a(7)^x$.*

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Step 3 Predict the e-mails after 1 week.

$y = 8(7)^x$ *Write the function.*

$= 8(7)^7$ *Substitute 7 for x (1 week = 7 days).*

$= 6{,}588{,}344$ *Use a calculator.*

There will be 6,588,344 e-mails after one week.

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4 Look Back

You chose the ordered pair (0, 8) to write the function. Check that every other ordered pair in the table satisfies your function.

y	$= 8(7)^x$
56	$8(7)^1$
56	$8(7)$
56	56 ✓

y	$= 8(7)^x$
392	$8(7)^2$
392	$8(49)$
392	392 ✓

y	$= 8(7)^x$
2744	$8(7)^3$
2744	$8(343)$
2744	2744 ✓

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Helpful Hint

You can choose any given ordered pair to substitute for x and y. However, choosing an ordered pair that contains 0 will often result in easier calculations.

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Check It Out! Example 3

Use the data in the table to describe how the oven temperature is changing. Then write a function that models the data. Use your function to predict the temperature after 1 hour.

Oven Temperature				
Time (min)	0	10	20	30
Temperature (°F)	375	325	275	225

1 Understand the Problem

The answer will have three parts–a description, a function, and a prediction.

2 Make a Plan

Determine whether the data is linear, quadratic, or exponential. Use the general form to write a function. Then use the function to find the temperature after one hour.

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3 Solve

Step 1 Describe the situation in words.

Oven Temperature	
Time (min)	Temperature (°F)
0	375
10	325
20	275
30	225

+ 10 + 10 + 10

– 50 – 50 – 50

Each 10 minutes, the temperature decreases by 50 degrees.

Step 2 Write the function.

There is a constant reduction of 50° each 10 minutes. The data appear to be linear.

$y = mx + b$ *Write the general form of a linear function.*

$y = -5(x) + b$ *The slope m is –50 divided by 10.*

$375 = -5(0) + b$ *Choose an x- and y-value from the table, such as (0, 375).*

$375 = b$

Step 3 Predict the temperature after 1 hour.

$y = -50x + 375$ *Write the function.*

$= -50(6) + 375$ *Substitute 6 for x (6 groups of 10 minutes = 1 hour).*

$= 75°$ F *Simplify.*

The temperature will be 75°F after 1 hour.

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4 Look Back

You chose the ordered pair (0, 375) to write the function. Check that every other ordered pair in the table satisfies your function.

y	$= -5(x) + 375$
325	$-5(10) + 375$
325	$-50 + 375$
325	325 ✓

y	$= -5(x) + 375$
275	$-5(20) + 375$
275	$-100 + 375$
275	275 ✓

4 Look Back

You chose the ordered pair (0, 375) to write the function. Check that every other ordered pair in the table satisfies your function.

y	$= -5(x) + 375$
225	$-5(30) + 375$
225	$-150 + 375$
225	225 ✓

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Lesson Quiz: Part I

Which kind of model best describes each set of data?

1.

Time (s)	Height of Ball (ft)
0	200
1	184
2	136
3	56

quadratic

2.

Value of Townhouse	
Age (yr)	Value ($)
0	100,000
1	102,000
2	104,040
3	106,121

exponential

Lesson Quiz: Part II

3. Use the data in the table to describe how the amount of water is changing. Then write a function that models the data. Use your function to predict the amount of water in the pool after 3 hours.

Water in a Swimming Pool	
Time (min)	**Amount of Water (gal)**
10	327
20	342
30	357
40	372

Increasing by 15 gal every 10 min; $y = 1.5x + 312$; 582 gal