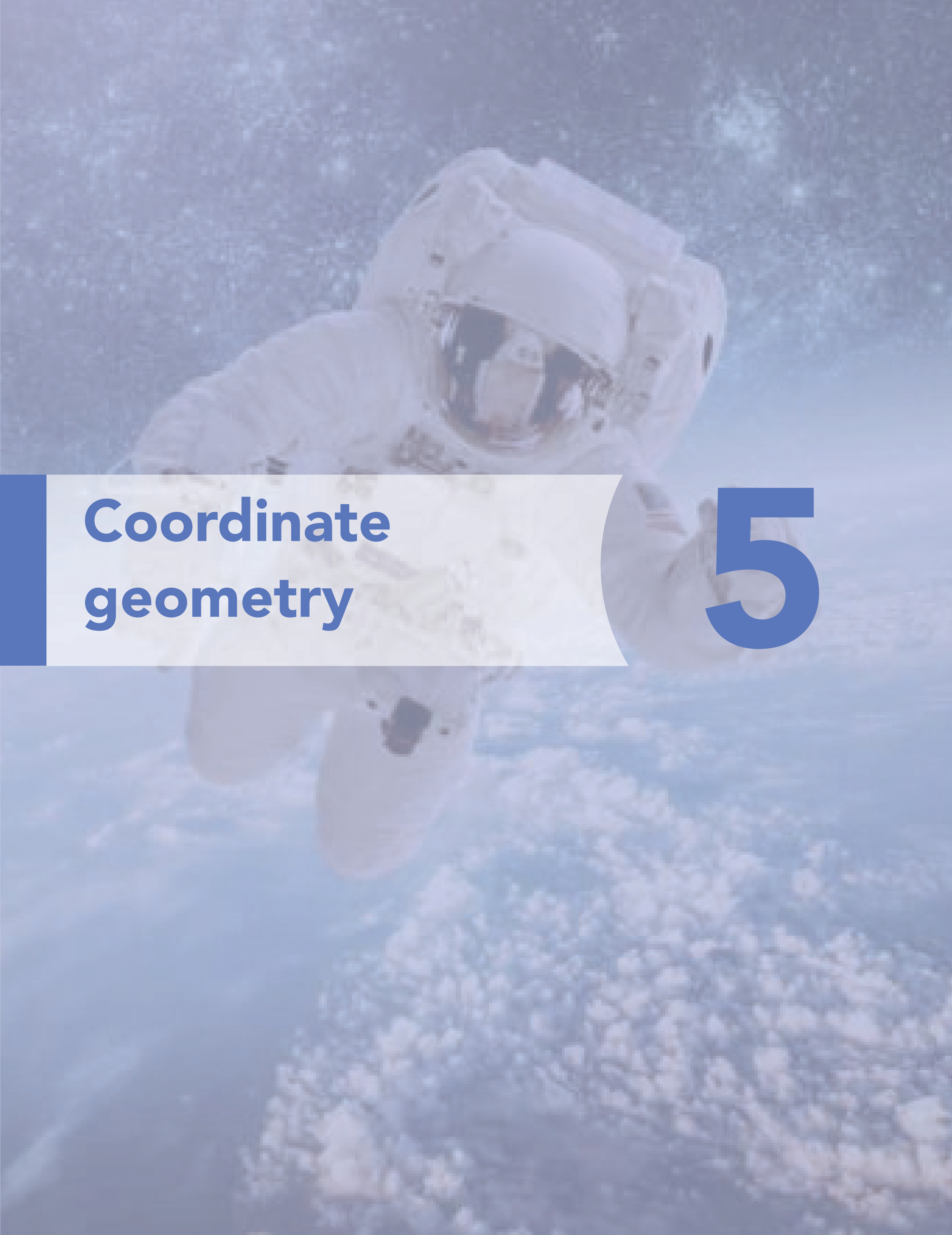
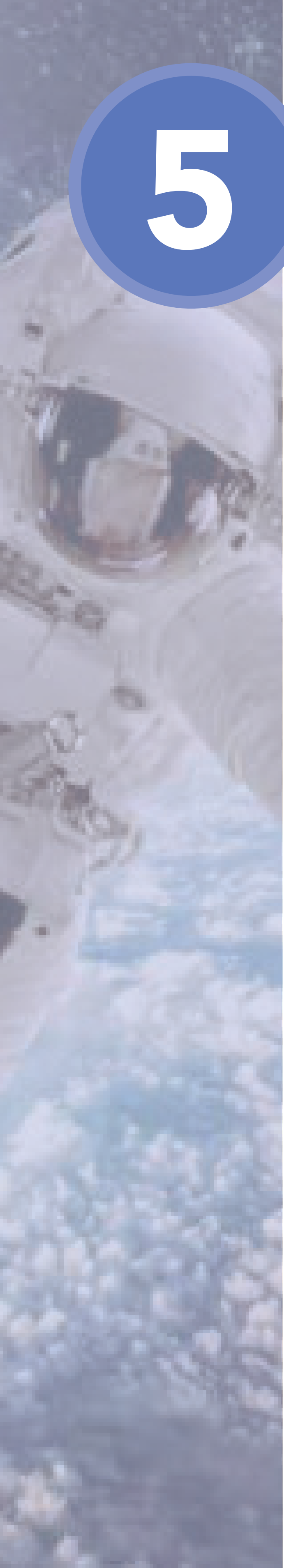


5 Coordinate geometry



5 Coordinate geometry

KEY CONCEPTS

Form

RELATED CONCEPTS

Change, Representation, Space

GLOBAL CONTEXTS

Orientation in space and time

Statement of inquiry

Forms in space help us to understand changes in representation of objects.

Factual

- What is an ordered pair in a coordinate system?
- What is the gradient of a straight line?

Conceptual

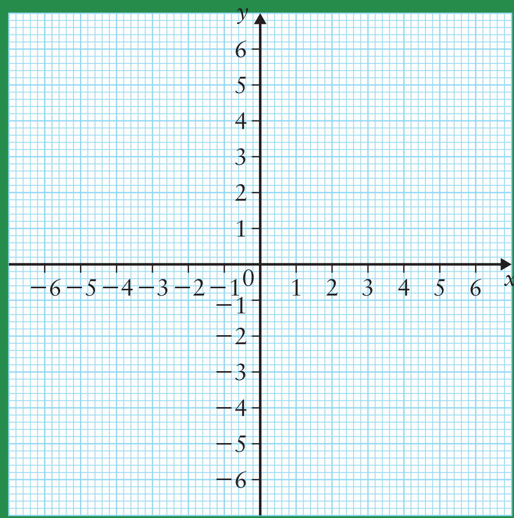
- How can you determine the equation of a straight line?
- How do you know whether two lines will intersect?

Debatable

- Can all straight lines be described in gradient-intercept form?
- Can two lines intersect at more than one point?

Do you recall?

- Do you remember how to represent a point with coordinates (x, y) in the number plane? Copy the number plane and plot these points: $A(3, 4)$, $B(-2, 3)$, $C(-2, 0)$, $D(5, -1)$.

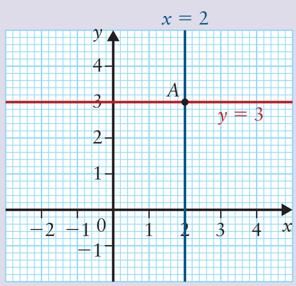


5.1 Points in the number plane

Explore

Look at the lines drawn in this diagram.

List all the information that the diagram shows. Include the coordinates of the intersection point, A.



Hint

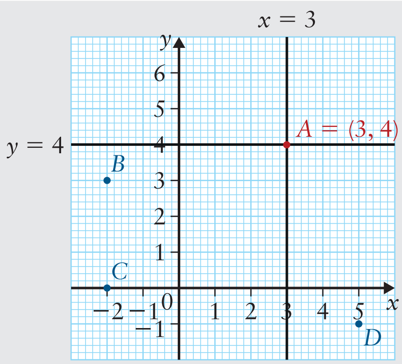
Ordered pairs are used to describe points in the coordinate plane. For example, the point (4, 3) has x-coordinate 4 and y-coordinate 3.

Worked Example 5.1

Plot the points $A(3, 4)$, $B(-2, 3)$, $C(-2, 0)$ and $D(5, -1)$ on the coordinate plane.

Solution

Point A is the point of intersection of the lines $x = 3$ and $y = 4$. You can find points B, C and D in a similar way.



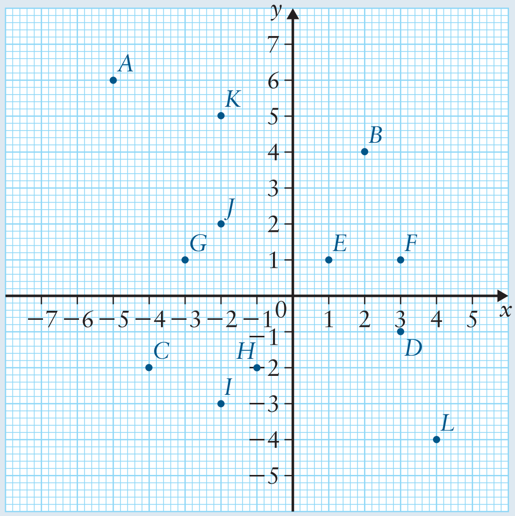
Fact

The x-axis can be described as the line $y = 0$ and the y-axis as the line $x = 0$.

5 Coordinate geometry

Coordinates are used to pinpoint locations on maps

Exercise 5.1



1 Use the diagram to write the coordinates of each of these points.

a *A* **b** *B* **c** *C* **d** *D* **e** *E* **f** *F*

g *G* **h** *H* **i** *I* **j** *J* **k** *K*

2 Using the diagram in question 1, name the points with:

a x-coordinate −2 **b** y-coordinate 1.

3 Draw a coordinate plane with x- and y-axes from −6 to 6. Plot each set of points and join them in the order given. Name each geometrical shape that you make.

a (−2, 2), (2, 2), (2, −2), (−2, −2),(−2, 2)

b (−5, 1), (−2, 1), (0, −1), (−6 ,−1), (−5, 1)

c (0, 1), (2, −2),(−3, −1), (0, 1)

d (−3, 4), (−1, 4), (−2, 2), (−4, 2), (−3, 4)

4 Find the distance between each pair of points.

a (2, 5) and (−3, 5)

b (−1, 4) and (−7, 4)

c (−3, 4) and (−3, −5)

d (4, 9) and (4, −1)

e (2, −2) and (5, 2)

Hint 4a

Plot the points and count the number of units between them.

Hint 4e

Use Pythagoras' theorem $a^2 + b^2 = c^2$.

Challenge e

5 Complete the tables by filling in the missing x- or y-coordinates on each of these lines.

Line AB

x	−3		3	
y		3		1

Line CD

x	−3		−1	
y		2		6

Line EF

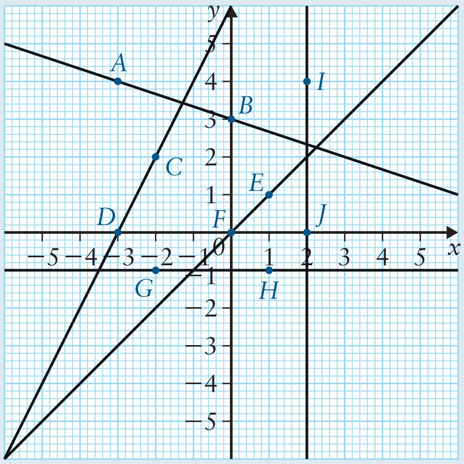
x	1		−3	
y		2		−5

Line GH

x	−4		0	
y		−1		−1

Line IJ

x	2		2	
y		0		−4



Challenge

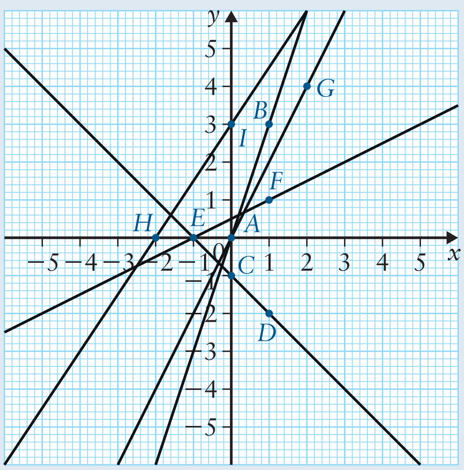
6 Match each of the lines in the grid below with the correct rule.

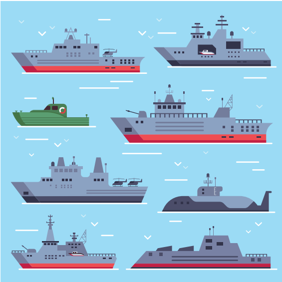
The x-coordinate is half of the y-coordinate.

The sum of the x- and y-coordinates is −1.

The y-coordinate is three times the x-coordinate.

The x-coordinate is 1 less than double the y-coordinate.





Connections

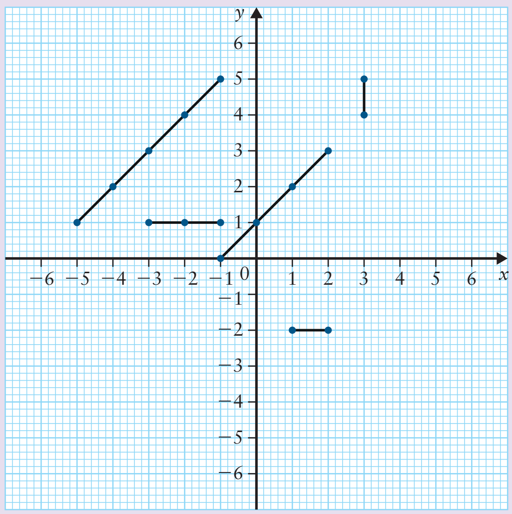
Fun with the number plane: sinking battleships!

This is a game for two players/groups.

1. Each player draws their fleet of ships on a coordinate grid by plotting points at the intersection of the gridlines. Agree on the size of the coordinate grid in advance – the bigger it is, the longer the game is likely to take. Each ship must be represented by a continuous row of points along either a horizontal, vertical or sloping line. Each person's fleet should consist of ships with 5, 4, 3, 2 and 2 plotted points.
2. The aim of the game is to 'sink' each other's ships by guessing where they are positioned on the grid.
3. Players take turns to play. Each player is given 2 'shots' on each turn. For each shot you should guess a pair of coordinates where you think your opponent has positioned a ship. The coordinates you try must consist of all combinations of sign, that is, one each of (+, +), (−, +), (−, −), and (+, −). A player who violates this rule forfeits the rest of the turn. You should record the shots that both you and your opponent make on separate charts so you know what has been hit.
4. If a shot misses, your opponent declares 'miss' and you both place an open circle in the appropriate position on your chart.
5. If a shot hits, your opponent declares 'hit' and you both place an X in the appropriate position.
6. Your opponent should tell you when you have found all the coordinates of an entire ship, by saying, for example, 'You sank a ship of size 3.'
7. Continue taking turns to play until one of you has sunk the other's entire fleet.

   Here is an example of how to draw a fleet of ships.

   Guessing (1, −2) is a hit and (0, 0) is a miss!



Explore

Imagine you are a carpenter. You have a straight edge, a pencil and a piece of wood. Before cutting the wood, you need to draw a line so that you can see where to use your saw. What do you need to know/do?

5.2 Graphing straight lines

The points on a straight line are linked by an equation. To graph a straight line, you need to plot at least two points on the line that satisfy this equation. Two points that can easily be found are the ***x*-intercept**, where the straight line crosses the x-axis, and the ***y*-intercept**, where the straight line crosses the y-axis.

Hint

You need at least two points to draw a straight line.

Worked Example 5.2

A straight line has equation $x + y = 4$. The table shows the x- and y-coordinates of the line.

Fill in the missing values in the table.

x	1		3		5
y		2		5	

Plot the points on a coordinate plane and join them to draw the graph of the line.

Solution

Understand the problem

A straight line is a set of points on coordinate plane. Identify each missing x- or y- coordinate. Plot the points and join them to draw the graph.

Make a plan

We need to substitute the given x- or y- coordinate into the equation of the line to figure out the missing ordered pair. When we have found all five ordered pairs on the line, we can plot and join them to graph the line.

Carry out the plan

To find the missing coordinate, substitute the known part of each ordered pair into the equation of the line.

When $x = 1$, $1 + y = 4$

$y = 4 - 1 = 3$

The first point is (1, 3).

When $y = 2$, $x + 2 = 4$

$x = 4 - 2 = 2$

The second point is (2, 2).

When $x = 3$, $3 + y = 4$

$y = 4 - 3 = 1$

The third point is (3, 1).

When $y = 5$, $x + 5 = 4$

$x = 4 - 5 = -1$

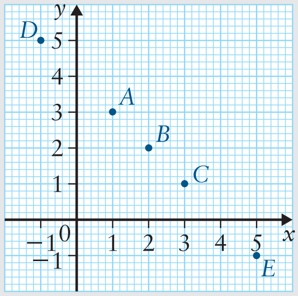
The fourth point is (−1, 5).

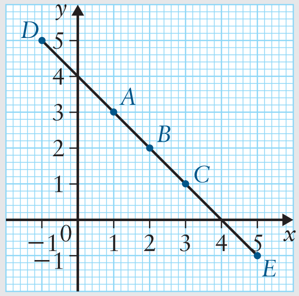
When $x = 5$, $5 + y = 4$

$y = 4 - 5 = -1$

The fifth point is (5, −1).

Now, plot these points $A(1, 3)$, $B(2, 2)$, $C(3, 1)$, $D(-1, 5)$ and $E(5, -1)$ on the coordinate plane and join them with a straight line. The line could be drawn by using any two of the points.





Look back

Is the solution true? Yes. When you add the x- and y-coordinates of the points A, B, C, D, E you get 4.

A: $1 + 3 = 4$, B: $2 + 2 = 4$, C: $3 + 1 = 4$, D: $-1 + 5 = 4$, E: $5 + (-1) = 4$

Worked Example 5.3

A straight line has the equation $x - y = -3$.

Identify 5 different points on the line. Plot and join the points to draw the graph of the line.

Solution

A straight line is a set of points on coordinate plane. We are asked to find 5 different points on the line $x - y = -3$.

The plan is to find 5 ordered pairs or points by using the equation of the line. We can select any 5 x-coordinates and find corresponding y-coordinates from the equation of the line. Then plot the points on the plane and connect them with a straight line.

To find the points, substitute selected x values into the equation and work out the corresponding y values.

When $x = 1$, $\quad 1 - y = -3$

$$1 - y + (-1) = -3 + (-1) = -4$$

$$y = 4$$

The first point is $(1, 4)$.

When $x = 2$, $2 - y = -3$

$2 - y + (-2) = -3 + (-2) = -5$

$y = 5$

The second point is (2, 5).

When $x = 3$, $3 - y = -3$

$3 - y + (-3) = -3 + (-3)$

$-y = -6$

$y = 6$

The third point on the line is (3, 6).

When $x = 4$, $4 - y = -3$

$4 - y + (-4) = -3 + (-4)$

$-y = -7$

$y = 7$

So, the fourth point is (4, 7).

When $x = 5$, $5 - y = -3$

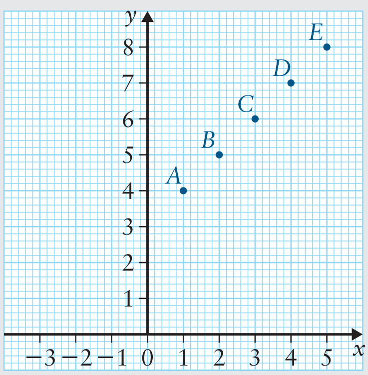
$5 - y + (-5) = -3 + (-5)$

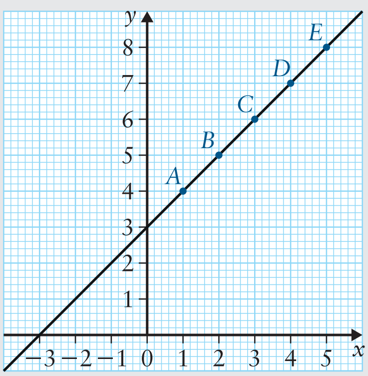
$-y = -8$

$y = 8$

The fifth point is (5, 8).

Plot the points $A(1, 4)$, $B(2, 5)$, $C(3, 6)$, $D(4, 7)$ and $E(5, 8)$ on the coordinate plane and join them with a straight line.





Does the answer fit the equation? Yes. When you subtract the y-coordinates from the x-coordinates of the points A, B, C, D you get −3.

A: $1 - 4 = -3$, B: $2 - 5 = -3$, C: $3 - 6 = -3$, D: $4 - 7 = -3$ and E: $5 - 8 = -3$

Reflect

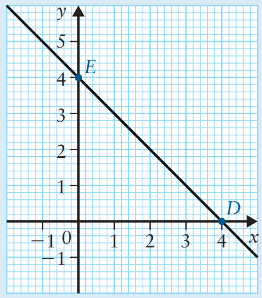
Think about each of these questions:

- Could you drawn the lines in Examples 5.2 and 5.3 differently?
- Is there a simpler method?
- Which method do you prefer?

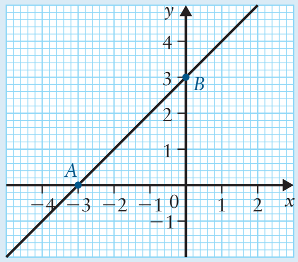
Hint

You only need to plot two points to draw a straight line. The x- and y-intercepts are easy to find because one of the coordinates is 0 at each of these points.

For Example 5.2, a simple way of drawing the line with equation $x + y = 4$ is to plot the x-intercept (4, 0) and the y-intercept (0, 4) and connect these points on the coordinate plane.



For Example 5.3, a simple way of drawing the line with equation $x - y = -3$ is to plot the x-intercept (−3, 0) and the y-intercept (0, 3) and connect these points on the coordinate plane.



Challenge

Hint

If a point is on a line, then the coordinates of the point must satisfy the equation of the line.

If you are given the equation of a straight line, how can you tell whether a point lies on that line? For example, how could you tell whether the point (1, 3) lies on the straight line with equation $y = 3x + 2$?

Worked Example 5.4

Does the point (1, 3) lie on the straight line with equation $y = 3x + 2$?

Solution

You want to know whether the point (1, 3) is on the straight line with equation $y = 3x + 2$.

You can substitute the coordinates of the point (1, 3) into the equation to see if it satisfies the rule.

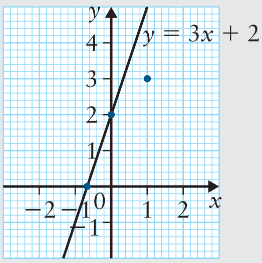
Now, substituting $x = 1$ into $y = 3x + 2$ gives

$$y = 3 \times 1 + 2$$

$$y = 5$$

The y-coordinate of the point (1, 3) is 3. Since $3 \neq 5$, the point (1, 3) is not on the line $y = 3x + 2$.

By looking back at the x- and y- intercepts of the line with equation $y = 3x + 2$ are (0, 2) and $\left(-\frac{2}{3}, 0\right)$ respectively. If you draw the line $y = 3x + 2$ and plot the point (1, 3) on a coordinate plane, you can see that this point is not on the line.



Exercise 5.2

1 Copy and complete the table for each of these equations.

a $y = 2x$

x	0	1	2
y			

b $y = 3x + 1$

x	0	1	2
y			

c $y = 2x - 1$

x	0	1	2
y			

d $y = 2 - 3x$

x	0	1	2
y			

2 On separate coordinate grids, draw the graphs of each of the equations in question 1.

3 Find the x and y-intercept of the graph of each of these equations.

a $y = 2x$

x	0	
y		0

b $y = 3x + 1$

x	0	
y		0

c $y = 2x - 1$

x	0	
y		0

d $y = 2 - 3x$

x	0	
y		0

4 On separate coordinate grids, plot the x and y-intercept for each equation given in question 3. Then join these points to graph each line.

5 Draw the graph of each of these equations by plotting the x and y-intercepts only.

a $x + 2y = 4$ b $y = 2x - 1$

c $y = \frac{x}{2}$ d $2x + y - 4 = 0$

6 On which of these equations does the point $(1, -2)$ lie? Show how you work out your answer.

a $x + y = 1$ b $2x - 3y = 6$

c $x - 2y - 5 = 0$ d $y = -2x$

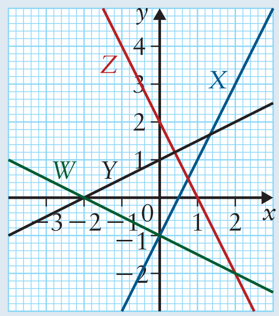
7 Match each of these equations to its graph.

a $y = \frac{1}{2}x + 1$

b $y = 2x - 1$

c $y = -2x + 2$

d $y = -\frac{1}{2}x - 1$



8 Write the x-intercept and the y-intercept of each of the straight lines (a, b, c and d) in question 7.

9 Which of the points $(1, 4)$ and $(-2, -2)$ lies on the line $2x - y + 2 = 0$? Show how you work out your answer.

5.3 Horizontal and vertical lines

Explore

- Plot the following points on a coordinate plane. Join them with a line. What do you notice?

Table 1

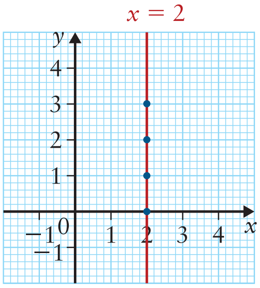
x	2	2	2	2
y	0	1	2	3

Table 2

x	0	1	2	3
y	3	3	3	3

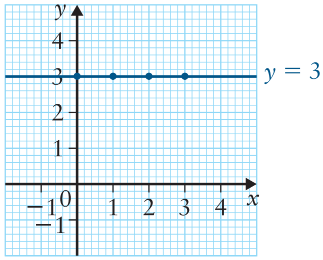
- How would you describe horizontal and vertical lines?

Vertical lines are described as $x = a$, where a is the intersection of the vertical line and x-axis.



Vertical line

Horizontal lines are described as $y = b$, where b is the intersection of the horizontal line and y-axis.



Horizontal line

Fact

The x-axis and the y-axis can be represented as horizontal and vertical lines respectively.

Horizontal and vertical lines

Worked Example 5.5

Draw the lines $x = 4$ and $y = -2$ on a coordinate plane. State whether the lines are horizontal or vertical.

Solution

We need to draw both the lines x = 4 and y = −2 on a coordinate plane by plotting points on the lines and connecting them.

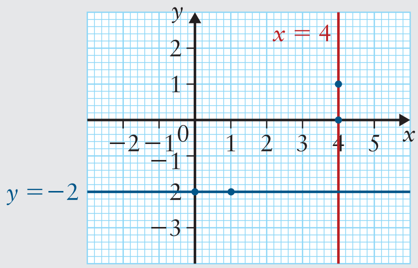
The plan is to plot a minimum of two points on each line and graph the lines by connecting these two points. We can use the definition of vertical and horizontal lines to identify them.

Now, (4, 0) and (4, 1) are two points on the line $x = 4$. (0, −2) and (1, −2) are two points on the line $y = -2$.

The line $x = 4$ has the form $x = a$, so it is a vertical line.

The line $y = -2$ has the form $y = b$, so it is a horizontal line.

Here are the graphs of the lines.



Looking back, $x = 4$ is a vertical line; the x-coordinate of all the points on the line is 4 and the line cuts the x-axis at 4.

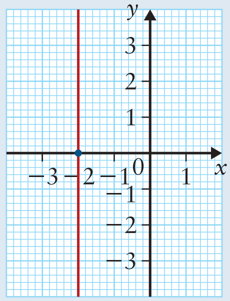
$y = -2$ is a horizontal line; all the points on the line have y-coordinate −2 and the line cuts the y-axis at −2.

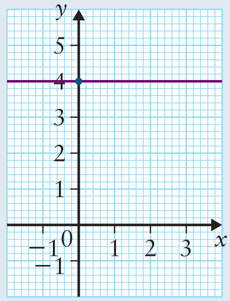
Reflect

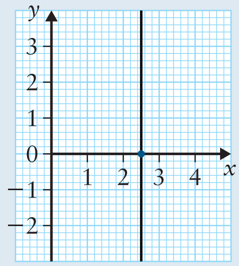
Discuss whether Worked example 5.5 could have been answered differently? If so, how?

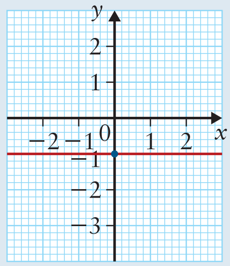
Exercise 5.3

1 Write down the equation of each of these vertical or horizontal lines.

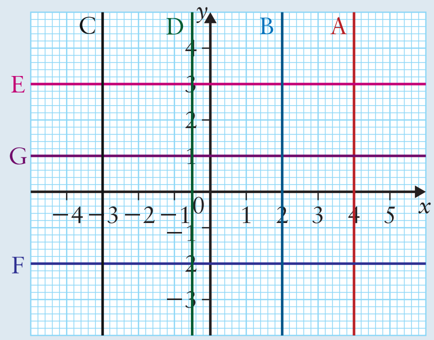
a


b


c


d


2 Write down the equation of each of the lines A to G.



3 Draw these vertical and horizontal lines on the same coordinate plane.

$x = 3$ $\quad x = -1$ $\quad y = -3$ $\quad y = -1$

What does the enclosed shape look like?

4 Write the coordinates of the points of intersection of all the lines A to G in question 2. How many intersection points did you count?

5 What would you call a line:

a parallel to the x-axis

b parallel to the y-axis?

6 Does the point (4, 2) lie on the line $x = 4$? What about $y = 3$? Explain how you know.

7 Find the point of intersection of each pair of lines.

a $x = 2$ and $y = 5$
b $x = -3$ and $y = 5$
c $x = -4$ and $y = -3$
d $x = 5$ and $y = -1$
e $x = 0$ and $y = 2$
f $x = -5$ and $y = 0$
g $x = 0$ and $y = 0$
h $x = -\frac{1}{3}$ and $y = \frac{1}{2}$

8 Find the point of intersection of each pair of lines.

Challenge

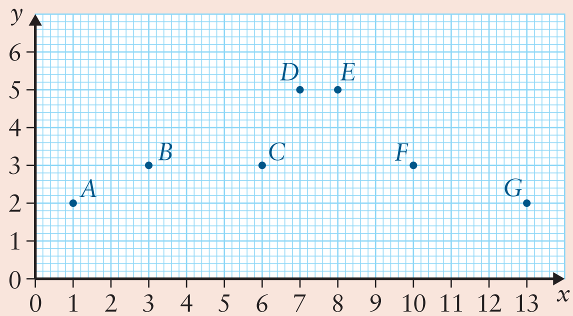
a $x + y + 1 = 0$ and $x = 2$
b $y = 3x - 4$ and $y = 2$
c $y = -\frac{2}{3}x$ and $x = 6$
d $2x - y + 3 = 0$ and $y = 1$
e $x = 0$ and $y = -x - 4$
f $y = 0$ and $y = \frac{1}{2}x - 3$

5.4 Gradient and equation of a line

5.4.1 Gradient

Investigate

The diagram shows part of a map. The letters represent towns. Jason and Kaan are travelling from A to G. Investigate the following questions in pairs or in small groups.



1 Find out which line segment out of each pair is the steepest.

a BC or DE
b AB or CD
c EF or FG

2 Identify if the path is sloping up, sloping down or not sloping for each of these routes.

a A to B
b B to C
c C to D
d D to E
e E to F
f F to G

3 Describe a rule for giving the steepness of any part of a route (AB, BC, etc.). Justify your rule.

Fact

The gradient of a line is also called **slope** of the line.

Road sign showing a gradient

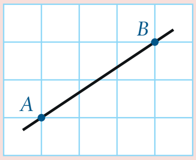
The **gradient** or **slope** measures the steepness of a line.

Gradient can be defined as the ratio of rise and run.

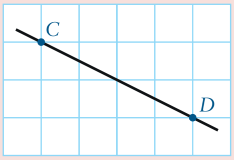
$$\text{Gradient} = \frac{\text{rise (change in } y)}{\text{run (change in } x)}$$

Fact

- If a line is horizontal, then there is no change in y and so the gradient is zero.
- If a line is vertical, then there is no change in x and therefore the gradient cannot be defined.
- A line sloping upwards from left to right has a positive gradient. Line AB has a positive gradient.

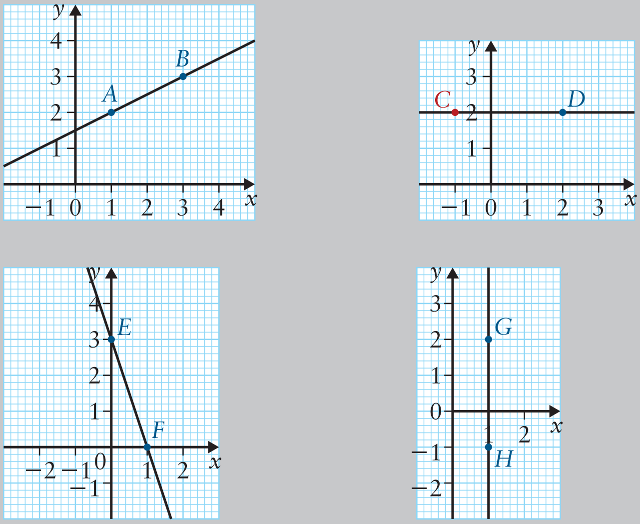


- A line that slopes downward from left to right has a negative gradient. Line CD has a negative gradient.



Worked Example 5.

Find the gradients of these lines.



Solution

We need to calculate the gradient of the lines AB, CD, EF and GH. We know the coordinates of two points on each line.

We use the formula for the gradient using the coordinates we know on each line. Apply the formula from left to right.

$$\text{Gradient of } AB = \frac{\text{change in } y}{\text{change in } x} = \frac{1 \text{ up}}{2 \text{ right}} = \frac{1}{2}$$

$$\text{Gradient of } CD = \frac{\text{change in } y}{\text{change in } x} = \frac{0}{3 \text{ right}} = 0$$

$$\text{Gradient of } EF = \frac{\text{change in } y}{\text{change in } x} = \frac{3 \text{ down}}{1 \text{ right}} = \frac{-3}{1} = -3$$

$$\text{Gradient of } GH = \frac{\text{change in } y}{\text{change in } x} = \frac{3 \text{ down}}{0} = \text{undefined}$$

Check that the calculated gradients make sense. Line *AB* has a positive gradient and ratio of rise: run is 1 : 2. Line *CD* is a horizontal line so you know the gradient is zero. Line *EF* has a negative gradient and the ratio of rise: run is −3 : 1. Line *GH* is a vertical line and the gradient of a vertical line cannot be defined.

Architectural design often requires an understanding of gradients (slopes)

Reflect

Can you find another, more direct way of calculating the gradient?

Fact

- The gradient or slope of a line is usually denoted by a lower-case letter m.
- If the coordinates of two points on a line are $A(x_1, y_1)$ and $B(x_2, y_2)$, then the gradient of the straight line *AB* can be described as $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ or $m_{AB} = \frac{y_1 - y_2}{x_1 - x_2}$

5.4.2 Equation of a straight line: the gradient–intercept form

Investigate

(Use available software or a GDC for this investigation)

1 On the same coordinate plane, sketch the lines with equation $y = mx + c$ for each pair of values for m and c given in Table 1.

Table 1

m	2	2	2	2
c	0	1	2	3

Hint

Go to https://www.geogebra.org and click on the *start calculator* button. Then you can enter the equation $y = mx + c$ for each pair of m and c values and you will be able to draw the graphs.

2 What do you notice about the graphs you drew in question 1?

3 Now use a new page to draw, on the same coordinate plane, the lines with equation $y = mx + c$ for each pair of values for m and c given in Table 2.

Table 2

m	0	1	2	3
c	1	1	1	1

4 What do you notice about the graphs you drew in question 3?

5 Sketch the straight lines given by each of the equations in Table 3. Identify the gradient and y-intercept for each line.

Table 3

Equation	Gradient	y-intercept
$y = x + 1$		
$y = 2x - 1$		
$y = x - 1$		
$y = 2x + 1$		
$y = \frac{1}{2}x - 2$		
$y = -3x + 4$		

6 Suggest a way to describe the relationship between the values of m and c and the graph of the equation $y = mx + c$. Justify your suggestion.

Fact

$y = mx + c$ is also called the **slope-intercept** form.

When the equation of a line is written in the form $y = mx + c$:

- m is the gradient
- c is the y-intercept.

$y = mx + c$ is called the gradient–intercept form of the equation of a straight line.

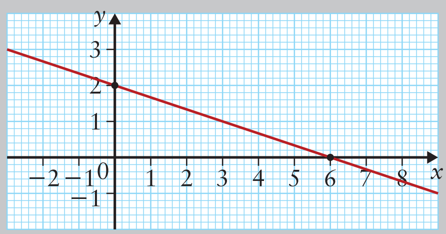
Worked Example 5.7

a For the line given in the diagram, find:

i the gradient

ii the y-intercept.

b Write the equation of the line in gradient–intercept form.



Solution

We need to identify the gradient and y-intercept of the graph. Notice that the line has a negative gradient.

The plan is to use the gradient formula with any two points on the line to find the gradient, m. We can find the y-intercept by looking at where the graph cuts the y-axis.

a **i** Two points on the line are the y-intercept (where the graph cuts the y-axis) (0, 2) and the x-intercept (where the graph cuts the x-axis) (6, 0). We can substitute these values into the gradient formula to find m:

$$m = \frac{2-0}{0-6} = \frac{2}{-6} = -\frac{2}{6} = -\frac{1}{3}$$

ii The y-intercept has coordinates (0, 2).

b In $y = mx + c$, m is $-\frac{1}{3}$ and c is the y-coordinate of the y-intercept, which is 2.

So the equation of the line is $y = -\frac{1}{3}x + 2$.

To check our solution, we can find the x-intercept by substituting in $y = 0$:

$$0 = -\frac{1}{3}x + 2$$

$$\frac{1}{3}x = 2$$

$$x = 6$$

This gives (6, 0) as the x-intercept, as required.

The y-intercept is where $x = 0$:

$$y = -\frac{1}{3} \times 0 + 2$$

$$y = 0 + 2$$

$$y = 2$$

This gives (0, 2) as the y-intercept. So both points satisfy the given graph.

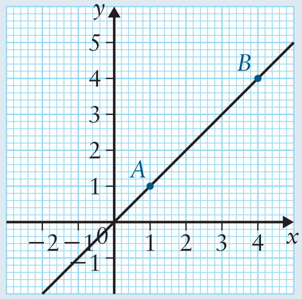
Reflect

Can the equation be found by a different method?

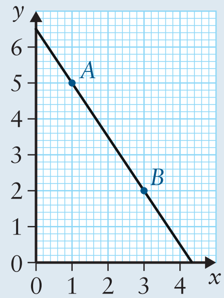
Exercise 5.4

1 Find the gradient of each line *AB*.

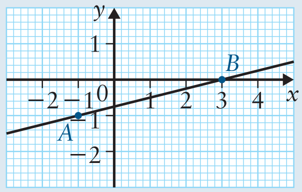
a



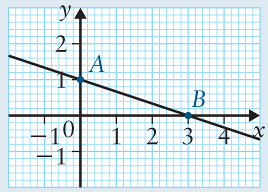
b



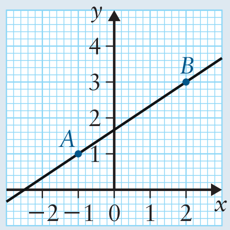
c



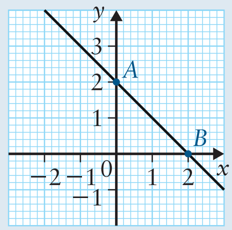
d



e



f

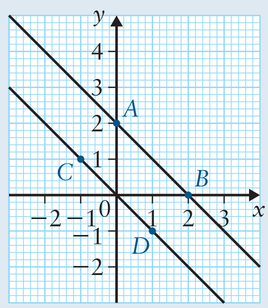


2 Find the gradient of the line passing through each pair of given points.

a $A(1, 1)$ and $B(2, 3)$ **b** $C(-1, 0)$ and $D(0, -1)$

c $E(3, 1)$ and $F(2, 4)$ **d** $G(0, 1.5)$ and $I(-1.5, 3)$

3 Find the gradients of the lines *AB* and *CD*. Are the lines parallel?



Fact

The equation of a straight line can be represented in different forms:

- gradient–intercept form: for example, $y = 2x + 3$
- general form: for example, $2x - y + 3 = 0$ or $y - 2x = 3$

Both of these forms represent the same graph.

Connections

See Chapter 6 for how to write an equation in different forms.

Hint

Parallel lines have the same gradient.

4 Identify the gradient (m) and the y-intercept (c) of each line.

a $y = 2x + 5$ b $y = x - 1$ c $y = -x + 3$

d $y = 5x - 1$ e $y = 3x$ f $x = 3$

g $y = 4$ h $3x - 2y = 4$ i $x - y + 1 = 0$

j $\frac{-1}{2}x - \frac{2}{3}y = 2$

Hint

Use the gradient-intercept form $y = mx + c$.

Challenge g

5 Draw the graph of each of these straight lines, given the gradient (m) and y-intercept (c).

a Gradient is -2 and y-intercept is 2

b Gradient is 3 and y-intercept is -2

c $m = 2$ and $c = 0$

d $m = -1$ and $c = 5$

e Gradient is $\frac{1}{2}$ and y-intercept is 1

6 Lines are parallel if they have the same gradient. Which of these pairs of lines are parallel?

a $y = 2x - 5$ and $y = 2x + 1$

b $y = -x + 3$ and $y = 1 - x$

c $y = 3x - 2$ and $y = 2x - 3$

d $x + y = 2$ and $-2x - 2y - 4 = 0$

Challenge d

7 Match each equation with the correct graph.

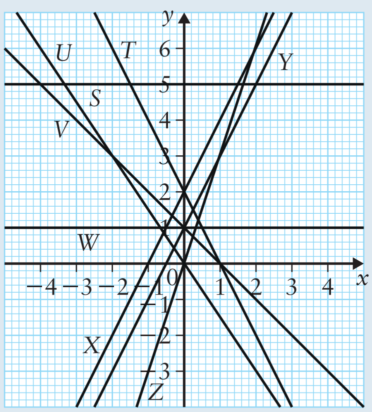
a $y = 2x + 1$

b $y = 3x$

c $y = -x + 1$

d $y = 5$

e $y = 2 - 2x$



5.5 Intersection of two lines

Explore

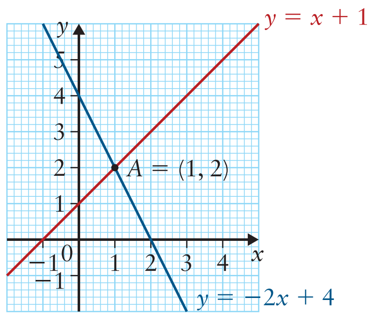
Can you answer the following questions?

1 In a coordinate plane, how do you know whether or not two lines will intersect?

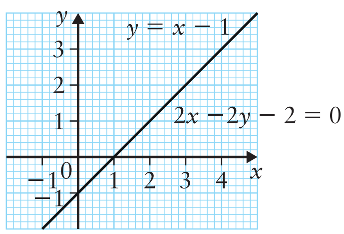
2 At how many different points can two lines intersect?

There are three different possibilities to describe whether two lines intersect.

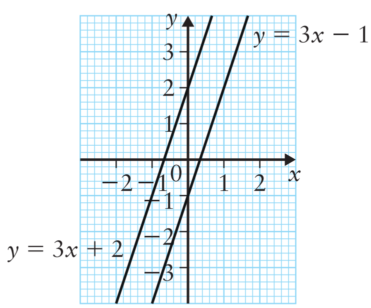
1 Two lines intersect at a point. For example, the lines $y = -2x + 4$ and $y = x + 1$ intersect at $A(1, 2)$.



2 Two lines intersect at every point: that is, they are the same line. For example, $y = x - 1$ and $2x - 2y - 2 = 0$ are the same line and they intersect at every point.

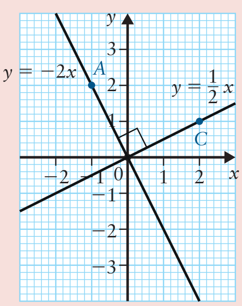


3 Two lines do not intersect at any point: that is, they are parallel lines. For example, $y = 3x - 1$ and $y = 3x + 2$ have no points of intersection because they are parallel.



Fact

If two lines are perpendicular (they cross at right angles), then the product of their gradients is −1.



Product of the gradients of the lines is $\frac{1}{2} \times -2 = -1$

Worked Example 5.8

Find the point(s) of intersection of each pair of lines.

a $y = x + 1$ and $y = 1 - x$

b $y = 3$ and $y = 2x + 1$

c $x + y = 2$ and $y = x$

Solution

For each pair of lines, we need to work out if there are any intersection points. We know how to draw the graph of a straight line from its equation.

To find the points of intersection, we can graph the lines using their equations and see if they intersect.

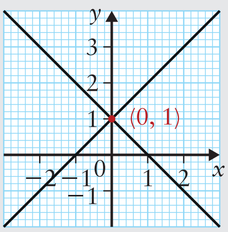
a The tables give three points on each line.

$y = x + 1$

x	−1	**0**	1
y	0	**1**	2

$y = 1 - x$

x	−1	**0**	1
y	2	**1**	0



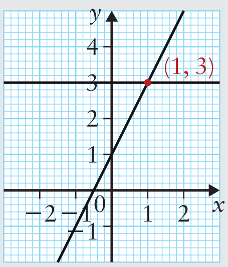
We can see from the table that (0, 1) is the point of intersection of the lines. The graphs of the lines $y = x + 1$ and $y = 1 - x$ are shown in the diagram.

b $y = 3$

x	−1	0	**1**
y	3	3	**3**

$y = 2x + 1$

x	−1	0	**1**
y	−1	1	**3**



The tables show that (1, 3) is the point of intersection.
The graphs $y = 3$ and $y = 2x + 1$ are shown in the diagram.

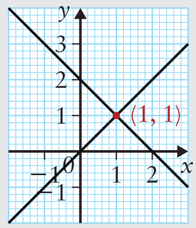
Intersecting airport runways

c $3x + y = 2$

x	−1	0	**1**
y	3	2	**1**

$y = x$

x	−1	0	**1**
y	−1	0	**1**



The tables show that (1, 1) is the point of intersection.
The graphs of $x + y = 2$ and $y = x$ are shown in the diagram.

The last step is to check whether our solutions make sense.
We can substitute the coordinates of each point of intersection into the equations of each pair of lines to see if the solutions are correct.

For $y = x + 1$ and $y = 1 - x$, substitute (0, 1):

$y = x + 1$, $1 = 0 + 1$, $1 = 1$ (true)

$y = 1 - x$, $1 = 1 - 0$, $1 = 1$ (true)

For $y = 3$ and $y = 2x + 1$, substitute (1, 3):

$y = 3$, $3 = 3$ (true)

$y = 2x + 1$, $3 = 2.1 + 1$, $3 = 3$ (true)

For $x + y = 2$ and $y = x$, substitute (1, 1):

$x + y = 2$, $1 + 1 = 2$, $2 = 2$ (true)

$y = x$, $1 = 1$ (true)

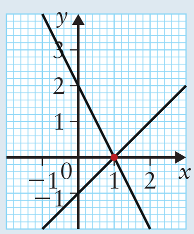
Connections

We can also find the points of intersection of two lines algebraically. We call this solving a system of equations or solving simultaneous equations.

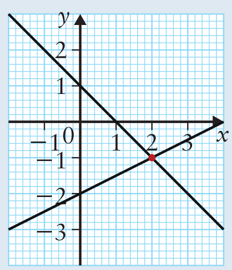
Exercise 5.5

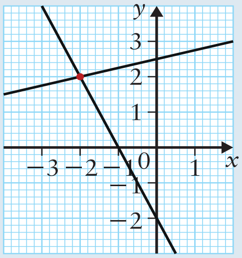
1 Find the point of intersection of each pair of straight-line graphs.

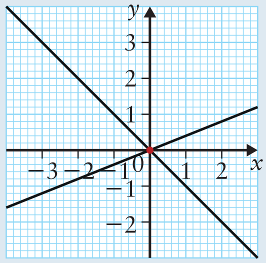
a



b



c 

d 

2 Find the intersection points of each pair of lines.

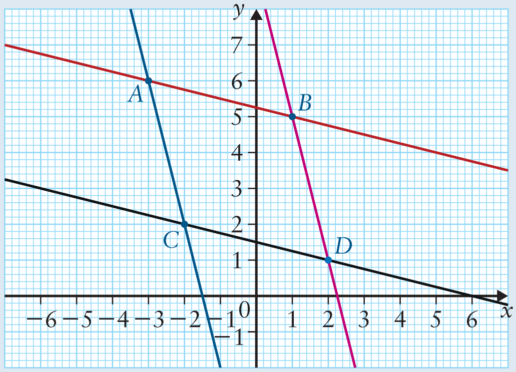
a $x + y = -2$ and $y = x$

b $2x - 3y - 6 = 0$ and $y = \frac{-1}{3}x + 1$

c $y = -x + 2$ and $x - 2y - 2 = 0$

d $y = 2x + 2$ and $y = 2x - 1$

3 The graphs of four straight lines are shown in the diagram.



a Find the point of intersection of each pair of lines.

i *AB* and *AC*

ii *AB* and *BD*

iii *CD* and *AC*

iv *CD* and *BD*

b What geometrical shape is *ABCD*? How do you know?

c Will *AC* and *BD* ever meet? Explain your answer.

Challenge b

Challenge c

4 Show that the lines $2x - 3y + 9 = 0$ and $-2x - 3y - 9 = 0$ intersect at $(-4.5, 0)$.

5 Find the points of intersection of the line $2x + y = 6$ with the x-axis and the y-axis.

6 Show that $x - 2y = 1$ and $y = 2x + 1$ are perpendicular lines.

7 Show that $2x - 4y + 6 = 0$ and $y = 1/2\,x - 1$ are parallel lines.

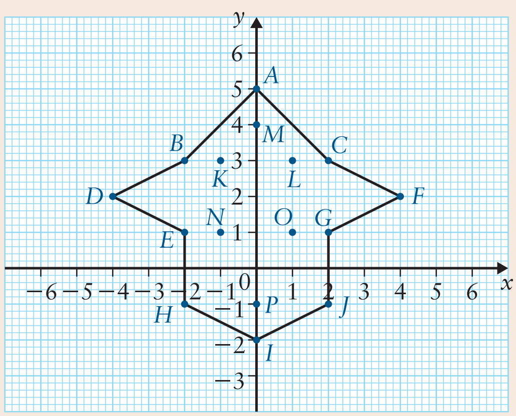
5 Coordinate geometry

Self assessment

- I can identify ordered pairs on a coordinate plane
- I can graph points on a coordinate plane
- I can find the distance between two points
- I can draw a straight-line graph on a coordinate plane
- I can find the x-intercept of a straight-line graph
- I can find the y-intercept of a straight-line graph
- I can determine whether or not a point lies on a straight line
- I can use the gradient formula
- I can use a GDC or available software to draw straight lines
- I can write the equation of a straight line in the form $y = mx + c$
- I can explain what is meant by the gradient–intercept form of the equation of a straight line
- I can use different forms of the equation of a straight line to draw its graph
- I can identify a horizontal line
- I can identify a vertical line
- I can represent the x-axis and y-axis as vertical and horizontal lines
- I can find the gradient of a straight line
- I can identify positive and negative gradients and describe them
- I can explain the steepness of a straight line
- I know that gradient and slope are the same thing
- I can explain whether or not two lines are parallel
- I can explain whether or not two lines are perpendicular
- I can find the points of intersection of two lines geometrically
- I can determine whether or not two lines will intersect

Check your knowledge questions

Questions **1** to **3** refer to this diagram.



1 Name the points whose coordinates are given by each of these ordered pairs.

a (0, 4) **b** (−2, 1) **c** (2, −1) **d** (2, 3)

Challenge e

2 Write down the coordinates of each of these points.

a N **b** H **c** F **d** B

3 Find the distance between each pair of points.

a M and P **b** D and F

c H and J **d** B and H

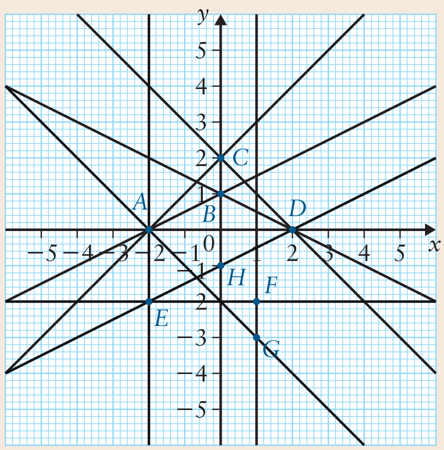
e A and F

4 Graph the straight line given by each of these equations.

a $y = 3x - 1$ **b** $y = -2x + 1$

c $y = 2x - 2$ **d** $x + y = 3$

Questions 5 to 8 refer to this diagram.



5 Find the equation of each line.

a AE **b** AD **c** BC **d** EF **e** FG

6 Find the gradient of each line.

a AB **b** AC **c** CD **d** BD **e** FG **f** EF

7 Find the equation of each line in gradient–intercept form.

a AG **b** AB **c** BD **d** CD **e** ED

8 Find the coordinates of the point of intersection of each pair of lines.

a AG and EF **b** AB and BD **c** AD and BC **d** AB and ED

e AG and CD

9 State the x-intercept and the y-intercept of each of these lines.

a $y = 3x - 5$

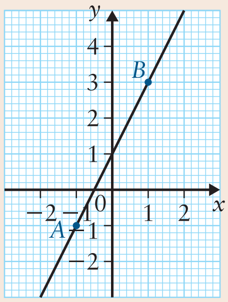
b $y = -x + 1$

c $x - y - 3 = 2$

d $2x + 3y = 12$

10 Find the equation of the straight line passing through points $A(1, 2)$ and $B(0, 3)$.

11 The graph shows the line AB.



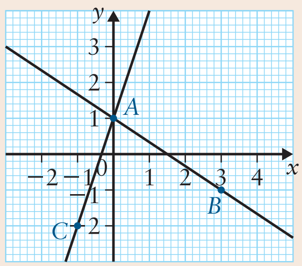
Find:

a the x-intercept

b the y-intercept

c the equation of the line in gradient–intercept form.

12 The diagram shows the lines AB and AC.



a Find:

i the equation of the line AB

ii the equation of the line AC.

Challenge b

b What do you observe about the lines AB and AC?

13 Determine whether each point lies on the given line.

a $A(1, 2)$ and $y = 2x - 1$

b $B(-1, 1)$ and $x + y = 0$

c $C(2, -1)$ and $x - 2y + 5 = 0$

d $D(0, 1)$ and $2x - 5y = -5$